The Cash Value of Conventionalism: Henri Poincaré on Geometry and Physical Relativity

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Poincaré's conventionalist doctrine of space, according to which the geometry of physical space is entirely a matter of definition, was widely contested by mathematicians and physicists during his lifetime. Following the discovery of the theory of relativity, Poincaré was led to defend anew his views on the foundations of geometry, and to acknowledge the success of Minkowski's theory of spacetime.

Poincaré's contributions to the theory of relativity are well known to historians, but it is not entirely clear how Poincaré understood the theory of relativity to impinge upon his doctrine of space. In part, at least, this is Poincaré's fault, as he did not express himself clearly on this topic. As a result, commentators have offered a wide variety of interpretations of the relation between Poincaré's conventionalist philosophy and his discovery and interpretation of the theory of relativity.[1]

In the process of elaborating the foundations of relativity, Poincaré recognized the notion of rigidity to be inconsistent with the new dynamics. Consequently, the Helmholtzian axiom of the free motion of rigid solids was unacceptable, and could no longer provide a foundation for geometry, even in a rough sense. The principle of relativity entailed for Poincaré replacing this axiom with that of the invariance of the propagation velocity of light. But did this replacement necessarily imply a modification of Poincaré's doctrine of space? Poincaré did not think so, as he explained in one of his last public lectures, at the University of London, on the fourth of May, 1912.

In the course of his lecture in London, Poincaré distinguished between Galilean covariance as a physical hypothesis, on one hand, and as a defini-

[1] For references, and an insightful comparison of Poincaré's and Einstein's philosophical approaches to relativity theory, see [Pat93].

tion of physical space and time, on the other hand. Undoubtedly, the same distinction applies in Poincaré's view to Lorentz covariance, which may be understood either as a physical law or as a definition of spacetime. Poincaré did not make this distinction explicit, however. By 1912, after some public hesitation, Poincaré had convinced himself of the experimental soundness of the new mechanics based on Lorentz covariance. But instead of defining space and time in virtue of Lorentz covariance, or via a protocol for clock synchronization (like Albert Einstein), he preferred to define space and time in virtue of Galilean covariance.

Poincaré explained that his point of view differed from that of certain unnamed physicists, who had adopted a spacetime ontology in the wake of Hermann Minkowski's theory. While Minkowski was not mentioned in Poincaré's lecture, Poincaré referred to a well-known result of Minkowski's paper, the identification of a class of events which can be neither the cause nor the effect of other given events. In Minkowski's terminology, such events are situated in spacelike regions of spacetime [Min08, §6]. This result, and others obtained by Minkowski and his followers, were instrumental in the spectacular rise to dominance of the spacetime approach to relativity theory from 1908 to 1911 [Wal99a, Wal99b]. In light of the success of Minkowskian relativity, Poincaré was understandably concerned for the future of his doctrine of the conventional nature of physical space.

References

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