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ABSTRACT

Validity of Inference

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Although it is generally agreed that an assertion in mathematics has to be proved in order to be warranted, there is no general agreement about what a proof is. In particular, if we try to say under what condition we are justified in asserting the conclusion of an inference, we are faced with a dilemma. On the one hand, the mere existence of a valid inference from premisses known to be true, is clearly not a sufficient condition. Nor is it enough to add merely that we have to show awareness of the inference by asserting the conclusion as following from the premisses; although admittedly this is how we usually announce an inference, we also require, metaphorically speaking, that the conclusion is "seen" to follow. On the other hand, it cannot be a necessary condition that we explicitly know the validity of the inference; if the validity of the inference had to be demonstrated before it could yield a ground for the conclusion, inferences could never be used to make epistemic advances.

Avoiding the horns of this dilemma, we should be able to specify a relation that a person needs to stand in to a valid inference in order to get a ground for its conclusion, when already having grounds for its premisses. We should then be able to derive that in fact she has a ground for the conclusion, given 1) that the inference is valid, 2) that she has grounds for the premisses, and 3) that she stands in the specified relation to the inference. To do that we need to have analyzed adequately what a valid inference is and what counts as a ground for assertions of various forms. This all is a philosophical project, which amounts to giving an account of how we make epistemic advances by inferences. The person who is in possession of a ground for an assertion need not bother about this project in order to make the assertion – she is justified in making the assertion in virtue of being in possession of a ground for the assertion, without having to show in addition that this is the case.

To carry out this project we cannot rely on the idea that the validity of an inference consists in its conclusion being a logical consequence of the premisses in the sense of Bolzano and Tarski. To get anywhere, we need a deeper analysis of what an inference is and what its validity consists in. It is suggested that an inference is to be seen as an act in which we transform grounds given for the premisses to a ground for the conclusion. An inference form is individuated by an operation that can be applied to grounds for the premisses, and is valid if the operation yields a ground for the conclusion when applied to grounds for the premisses. What is to count as grounds is only exemplified in this lecture, but can be developed in detail; formally, it becomes a kind of a so-called Curry-Howard correspondence.

This is in outline a way to solve the problems stated here. The relation that a person has to have to an inference in order to get a ground for the conclusion can now be specified by saying that she has to perform the inference, taken in the deepened sense now explained, that is, she has to apply the operation in question to the given grounds for the premisses. Given that the inference is valid, she then gets a ground for the conclusion as a result, because that is what it means for the inference to be valid as now defined; being in possession of a ground for the conclusion, she is justified in asserting the conclusion, without knowing that the inference is in fact valid. But, of course, by reflecting upon the inference act, she can get to know that the inference is valid.

Equipped with a notation for grounds, we may define a proof as a verbal expression denoting a ground. When the notion of proof is understood in this way, a proof of a theorem will encode operations by which, so to say, one sees the truth of the theorem.